Flight Efficiency in European Airspace

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Outline

1. The Problem
2. Data analysis algorithms
3. Optimisation strategies
   - Discrete approach
   - Continuous problem approach
The Problem

- Busy European airspace
- Civil aircraft routing inefficiencies:
  - Military airspace
  - Waypoints
- Investigate optimisation strategies
  - Flexible time access to military airspace
  - Modification of military airspace
  - Quantification of benefits associated with each strategy
Sample data:
- Over 2,000 European airports
- Circa 33,000 flights from a single day (departure airport/time, arrival airport/time, cruising altitude)
- Military airspace (location, altitudes)
Cost

- Costs based on path flown
- Fuel costs
- Delay costs
Data analysis algorithms

Data: Number of intersections

- Plot great circle of flight routes, assume uniform speed and height
- Plot paths of intervals of half an hour
- Assume all military airbases are available
- Work out number of intersections with each military airbase in that half an hour
- Plot flights paths for a given day: movie

To represent data we colour each airbase according to number of intersections; see movie!
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Moving airbases

- Investigate effect of moving one airbase by small amount of latitude/longitude
- Corresponds to moving airbases by about 30km along compass points as seen below
Optimal routing using actual waypoints/segments

Flights from LHR avoiding military space (blue):
Optimal routing using actual waypoints/segments

Flights from LHR allowing access to all military space (red):
And now on the whole of Europe . . .

Method explicitly outputs a cost saving of $30 per flight.
Discrete path planning

- What is the cheapest path from A to B?

- Dijkstra’s algorithm (or A*)
  - How to calculate gains from removing any one obstacle without having to re-run the search for each case?
Data analysis algorithms

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Linear programming

The shortest path problem is

$$
\min_{x_{i,j,f}} \sum_{i,j} C_{i,j,f} x_{i,j,f} \quad (1)
$$

subject to:

$$
\sum_{j} x_{i,j,f} - x_{j,i,f} = \begin{cases} 
1, & \text{if } i = \text{departure airport for flight } f, \\
-1, & \text{if } i = \text{arrival airport for flight } f, \\
0, & \text{otherwise.}
\end{cases} \quad \forall \ i, f. \quad (2)
$$

$$
x_{i,j,f} \geq 0, \ \forall \ i, j, f. \quad (3)
$$
Shortest Path on Visibility Graph – Toy Examples
Visibility Graph for Europe Data
Continuous pathfinding approach

Good initial guesses

Bad initial guesses
Continuous pathfinding approach

Curve $\gamma : [0, 1] \rightarrow \mathbb{R}^2$, such that $\gamma(0) = 0$, $\gamma(1) = x$,
Cruise speed $v_c$, Wind velocity field $w : \mathbb{R}^2 \rightarrow \mathbb{R}^2$.
Parametrisation: $\gamma(t) = tx + a(t)x^\perp$, $a(0) = a(1) = 0$.

$$\text{Time}[a] = T[a] = \int_0^1 \frac{\sqrt{1 + \dot{a}(t)^2}}{v(t, a, \dot{a}, w)} \, dt,$$

where

$$v(t) = \left( |v_c|^2 + 2(x + \dot{a}(t)x^\perp) \cdot w(tx + a(t)x^\perp) - |w(tx + a(t)x^\perp)|^2 \right)^{1/2}.$$

Euler-Lagrange equation: subject to $a(0) = a(1) = 0$.

$$\frac{d}{dt} \left( \frac{\dot{a}(t)}{v(t)\sqrt{1 + \dot{a}(t)^2}} - \frac{2x^\perp \cdot w(tx + a(t)x^\perp)}{v(t)^3} \right) + \frac{2(x + \dot{a}(t)x^\perp - w(tx + a(t)x^\perp)) \cdot \nabla w(tx + a(t)x^\perp) \cdot x^\perp}{v(t)^3} = 0.$$
Future ideas

- Find the path of several flights at once
- Include sector capacity constraints
- Include scheduling constraints
- Last two options require to include time in the optimisation.
- Optimize overall network efficiency.

Questions?
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